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THE DEBT OF MATHEMATICS TO THE CHINESE PEOPLE[1]

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TO write a satisfactory account of the contributions of the Chinese to the development of mathematics has always been a task beset with peculiar difficulties. These difficulties consist in the extreme uncertainty which the historian always feels as to the date and authenticity of any Chinese manuscript, coupled with the known tendencies of the Chinese people to glorify their ancestors and to exaggerate greatly the discoveries which they had made, in many cases attributing to them ideas and concepts which it is quite impossible that they can in reality have known.

As an illustration of these difficulties, let us mention two interesting examples where results obtained by Chinese mathematicians, and claimed as original discoveries by them, can with little short of absolute certainty be traced back to Greek influence and indeed to the particular influence of the greatest of the Greek mathematicians, Archimedes. First, we find attributed by later Chinese writers to a Buddhist monk Hang a proof that the number which we write 3^{361} is not infinite, but can be expressed either in words or in writing. To do this, Hang introduces a system of "octads," that is, groups of 10^8 successive integers, in such a way that the "first octad" consists of the integers from 1 to 10^8, the second octad of the numbers from 10^8 to 10^{16}, and so on. This scheme obviously permits a simple expression to be found for integers even much larger than 3^{361}, the number with which Hang was concerned. This procedure, as a moment's thought will convince us, is but one of an unlimited number of possible means of accomplishing the same object; and it is precisely the method adopted by Archimedes in his "Sand-reckoner," which makes it seem certain that the Chinese work was inspired by the result previously obtained by the great Syracusan.

[1] This article is based on a communication made to the society "Mathesis" by Professor Gino Loria, and published in the *Bollettino della Mathesis* in April, 1920. Through the kindness of Professor David Eugene Smith the communication came to my hands, and I am glad to take this means of presenting it to the notice of Americans who are interested in the history of science. Professor Loria's communication has been much condensed, and accordingly he is not responsible for the form of this article, but only for its content.—R. B. McClenon.

As our second illustration, we find a remarkably accurate approximation to the value of π worked out by Tsu Ch'ung-Chih in the 5th century. By working with a circle of radius 10^8 (it is significant that this is the same number that had been chosen by Hang as the basis of his system of "octads") Tsu found that π lies between 3.1415926 and 3.1415927. He accordingly suggested a better approximation for π than the familiar 3 1/7, namely 355/113 (which is equal to 3.1415929+). We have here an extension of the results given by Archimedes in his "Measurement of the Circle," and if we accept P. Tannery's[2] emendation of a passage in Heron's *Metrica,* Archimedes had further obtained the result,

$$\frac{195882}{62351} > \pi > \frac{211872}{67441}$$

Writing these values in the form of continued fractions,

$$3+\cfrac{1}{7+\cfrac{1}{16+\cfrac{1}{8+\cfrac{1}{1+\cfrac{1}{60}}}}} > \pi > 3+\cfrac{1}{7+\cfrac{1}{16-\cfrac{1}{31+\cfrac{1}{2+\cfrac{1}{9}}}}}$$

we are led to the approximation $\pi = 3 + \frac{1}{7+1/16} = \frac{355}{113}$, which could thus have easily reached China along with the other work of Archimedes. And even if we grant that Tsu may have originated this particular approximation for π, it still remains certain that the *method* which he used was not original with him, and it is almost a certainty that he obtained this method from Archimedes's famous work.

These illustrations, with many others of the same kind that might be cited, give rise to legitimate doubts whether in other cases, even where no direct connection can be established with other countries, the Chinese writers did not also receive their inspiration from abroad. These doubts are not quieted when we observe that practically all of the Chinese mathematical works are merely collections of problems, solved by rules which are not demonstrated; so that it is very difficult to determine whether these rules were really understood by the writer or not. What shall we say when we find in the midst of a set of trivial or even nonsensical problems one like the following? "There is a group of objects whose number is unknown; dividing by 3, the remainder is 2; by 5, the remainder is 3; and by 7, the remainder is 2; what was the number?" The author of the collection in which this occurs, Sun Tsu, obtains a solution, 23, by using a method called Ta Yen, which does not differ from that taught by Gauss[3] for solving similar problems in general: and what more significant praise could be given? But as Sun Tsu gives not the slightest indication that he regards this problem as more interesting or valuable than its trivial companions, we may well question his claim to the discovery of Ta Yen.

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[2] Memoires scientifiques, t. 3, Paris, 1915, p. 149.

One of the classical Chinese mathematical productions was the "Arithmetic in Nine Sections," but unfortunately nothing is known definitely as to the date at which the collection took its present form. Parts of it very likely go back to the period before the "burning of the books" (by the emperor Shih Hoang-ti) in 213 B. C. but much of it is certainly of later date. The following interesting and original problem is found in this collection:

"A square city of unknown side is crossed by a street which joins the centers of the north and south sides; at a distance of 20 paces north of the north gate is a tree which is visible from a point reached by going 14 paces south from the south gate and then 1775 paces west. What is the length of each side?"

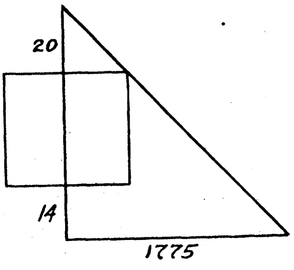

FIG. 1

The Chinese writer gives correctly the equation to which this problem leads, viz., $x^2+34x-2\cdot20\cdot1775=0$, but his statement as to how to solve the equation has probably been corrupted by copyists, as it merely states that the root of a certain expression must be found. As a matter of fact, the only positive root of this equation is $x=250$; which makes it clear that the originator of the problem must have chosen the numbers involved deliberately, so as to produce a simple result.

[3] *Disquisitiones Arithmeticae,* Sec. 32-36.

Whether the originator was a Chinese at all seems questionable, after examination of another problem of a similar kind which is met with in a work composed much later, namely, the "Nine Sections of Mathematics," written in 1247 by Ch'in-Chiu-Shoa. The problem reads:

"A circular castle has four gates facing the cardinal points; at a distance $a(=3)$ to the north of the north gate is a tree, which is visible from a point situated at the distance $b(=9)$ to the east of the south gate. What is the diameter of the castle?" Ch'in says this leads to the equation

$$1x^{10}+7ax^{8}+8a^{2}x^{6}-4(b^{2}-a^{2})x^{4}-2b^{2}\cdot 8a^{2}x^{2}-2b^{2}\cdot 8a^{2}b=0$$

and states that the root is $x=9$, the length of the diameter.

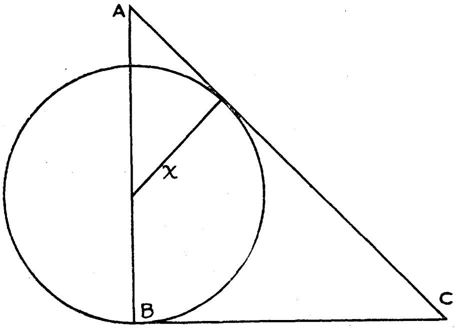

FIG. 2

Before commenting on this, let us solve the problem for ourselves. Thus, taking x as the radius, and expressing the hypotenuse of the right triangle ABC in two ways, we have

$$\overline{AC}^{2}=b^{2}+(2x+a)^{2}=(b+\sqrt{a^{2}+2ax})^{2}$$

from which we obtain $2x^{3}+ax^{2}=ab^{2}$,
an equation enormously simpler than that given by Ch'in. And not only that, but the result stated by Ch'in, $x=9$, does not satisfy his equation at all! Hence even if there is some copyist's error present, we seem forced to the conclusion that Ch'in did not very fully understand the problem, else he could not have supposed that it required so formidable an equation to solve it. Hence the doubt just expressed as to the Chinese origin of the problem. Moreover, no information whatever is given as to how an equation of this kind is to be solved. Presumably the method was similar to the one used in another case, which is more fully reported by Ch'in; but which also leaves much to be desired in the way of clearness and completeness. This other problem is as follows:

$$-x^{4}+763{,}200x^{2}-40{,}642{,}560{,}000=0.$$

"The first figure of the root is seen to be 8" we are told; but how this important fact "is seen" we are not informed. This significant omission arouses suspicion that the answer may have been known beforehand, as in the case of other problems referred to above. Knowing that a root of the equation lies between 800 and 900, the process known as Horner's method enables us to find a new equation whose roots shall be 800 less; and without going into the details of the computation it is sufficient to say that the transformed equation is

$$y^4+3200y^3+3,076,800y^2+826,880,000y-38,205,440,000=0,$$

and this way may be taken to be the meaning of the following table, which Ch'in gives in connection with his explanation of the problem:

8	root
38,205,440,000	absolute term
— 82,688	“ of 1st degree
— 30,768	“ “ 2nd “
— 32	“ “ 3rd “
— 1	“ “ 4th “

Barring the omission of zeros, which we must attribute to a copyist's error, this result coincides with the correct one given just above. But we still have too meager information as to the details of the work for us to be able to affirm confidently that Horner's method was known to the Chinese in the 13th century; we can only say that this method, or one practically identical with it, was known at that time, and we must await further evidence before affirming or denying the priority of the Chinese in its discovery.

Biernatzki[4] quotes from Ch'in Chiu-shao another illustration of the method, which however, has been so badly mangled by copyists, that we can not judge as to whether Ch'in was really able to solve the equation in question or not.

It is thus clear that much further investigation will be necessary before this interesting and important question in the history of mathematics, the question as to how much the progress of the science in reality owes to the Chinese people, can be answered with any degree of certainty. It is to be hoped that scholars qualified by knowledge of the language will in the near future enter upon these researches and carry them through to a successful conclusion.

Crelle's Journal, 52.